

## Mathematics Education Qualifying Exam January 2018

Instructions: The following questions constitute the Mathematics Education Qualifying Exam for January of 2018. The questions are separated into two sections; the first section is based on MTH 761 and the second section is based on MTH 762. You must answer two of the three questions in Section I as indicated (first question and then one of the remaining two) and two of the three questions in Section II. In both sections, make sure it is clear which questions you are answering and would like considered. You must pass each section with a 70% score in order to pass the exam. You have four hours to complete this exam. Remember to save your work frequently. The papers by Larsen (2013) and Zazkis and Liljedahl (2004) are provided on the USB flash drive as well as in print form for your convenience. In addition, for your convenience, hard copies of Pirie & Kieren's (1994) model along with two of the activities from MTH 761 are also provided. The video case of the Border Problem is on the flash drive, should you choose to respond to question 2 of section 1 and the vignette, *Not Quite Right*, is given in both print and electronic form.

### Section I:

1. One of the articles distributed for reading in Math 761 with regard to the teaching and learning of abstract algebra was Larsen's (2013), *A local instructional theory for the guided reinvention of the group and isomorphism concepts*. This article is provided in both electronic and print forms for your convenience.
  - a. Pirie & Kieren (1994) describe a general model for the development of mathematical understanding (we referred to it as an onion) with multiple layers. Attached is a graphic of the model and its stages. Compare and contrast the five-stage description of Larsen's Local Instructional Theory (LIT) and Pirie & Kieren's model.
  - b. While Pirie & Kieren provide a general model for the development of understanding, Kaput, Blanton, and Moreno (2008) provide a model for the development of *symbolic* meaning. Compare and contrast Kaput, Blanton, and Moreno's model with Larsen's LIT.
  - c. In class, we used guided discovery to explore some of the same ideas described in Larsen's (2013) guided reinvention approach using the two activities, *Form & Function*, and *Groups & Isomorphisms*. Compare and contrast the two approaches (LIT's Guided Reinvention and our classes' Guided Discovery). In particular, discuss the differences in how the basic moves on the triangle were expressed in each situation. For your convenience, I have provided copies of our two activities from class.

Choose and respond to one of the next two questions. If you choose to respond to both in some way, make sure you indicate which of the two questions you would like to be scored as your second question for the exam.

2. Consider the attached vignette, *Not Quite Right*, from a high school algebra class where the teacher is giving instruction on the relationship between the equation of a line and the equations of lines parallel and perpendicular to it. Give your responses to the questions that follow.
  - a. In class we discussed the five types of discourse routines (re-voicing students' contributions, orienting students to each other, pressing for explanations, connecting students' ideas, and making the structure of the discourse visible). Describe each of these routines and their role in maintaining an active mathematics classroom.
  - b. As we discussed in class, the teacher should use these discourse routines to center the focus on the students' thinking and work to establish and support productive mathematical discourse. In what ways did the teacher in the vignette, *Not Quite Right*, succeed or fail to accomplish these goals? Explain your thoughts citing specific instances. Particularly, in looking at the discourse routines listed in part (a), identify where in the vignette the teacher employed them. Be specific and cite instances along with how the student(s) responded.
  - c. Recall the video excerpts of the *Border Problem* taught by Cathy Humphreys from our class assignments (video is included on the flash drive for your reference). Compare and contrast the discourse routines used by Laura from the *Not Quite Right* vignette with those used by Cathy in the *Border Problem* video case. Discuss the types of questions asked by each teacher and the responses they elicited from the students. In particular, relate these discourse routines to Kaput, Blanton, & Moreno's model for the development of symbolic meaning.

3. In Math 761, we examined the following question: Consider the matrix,  $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ .

Find  $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1}$  by hand documenting your process. At this point, most people took

a row reduction approach after augmenting the matrix with a  $2 \times 2$  identity matrix. When pressed further as to why this method worked, the class realized that it was “just because that is what our teacher told us”. Recall that at this point, I asked the class to come up with transformation matrices corresponding with each stage of the row reduction process. When we finished, we multiplied these transformation matrices together to reveal the inverse matrix.

- a. Sfard (1991) refers to an operational-structural duality of mathematics as opposed to a procedural-conceptual dichotomy. Describe how this duality view of mathematical ideas is illustrated by the exploration we conducted in class with the inverse matrix process.
  
  
  
  
  
  
  
  
  
  
- b. Sfard (1991) also proposes a developmental model of *interiorization-condensation-reification* as students develop mathematical concepts. Describe this process and relate it to the inverse matrix example above.
  
  
  
  
  
  
  
  
  
  
- c. Sfard and Linchevski (1994) suggest that a process of reification must occur as students move from an operational conception to a structural conception. Discuss how technology (such as a CAS) could be helpful in the reification process for the inverse matrix situation described above.

**Section II: Choose two of the three items below for this portion of the exam.**

1. Consider the paper of Rina Zazkis and Peter Liljedahl. When designing a research study in mathematics education, there are many facets a researcher must consider prior to conducting the study including: clear research questions, well- chosen variables, carefully selected research methods, a plan that must address reliability and validity in the data collection and analysis, and a thoughtful procedure for analyzing the data. If you were to conduct a modified form of this study for your dissertation, what are some of the facets that you would address differently than Zazkis and Liljedahl? Make sure to provide solid reasons for your responses.

2. Recall Schoenfeld's list of *Standards for Judging Theories, Models and Results*. Use the seven standards given in his list while you examine *Understanding Primes: The Role of Representation* by Rina Zazkis and Peter Liljedahl. Also, where, if any, was there evidence or statements in this paper that addressed Schoenfeld's question, "How much faith should one have in a particular result?". Include a summary/closing paragraph.

3. Rina Zazkis and Peter Liljedahl (2004) wrote what kind of paper? Discuss, in detail, the methodology and instruments used for this study. Discuss what these scholars learned from the study and explain how they can make these claims by identifying the evidence they cite. Make sure your discussion goes beyond what is found in the abstract and provide evidence from the paper. Include a summary/closing paragraph.