Mathematics Education Qualifying Exam
January 2010

The following six questions constitute the Mathematics Education Qualifying Exam for January of 2010. The questions are separated into two sections; the first section is based on MTH 762 and the second section is based on EDU 614. You must answer two of the three questions in each section. Make sure it is clear which questions you are answering. You have four hours to complete this exam. Remember to save your work frequently.

Section I:

1. Three of the main learning theories in mathematics education are constructivism, sociocultural theory, and the emergent perspective (also referred to as social constructivism). Based on each of these different perspectives, discuss how you think an instructor would conduct the classroom discussion/activities differently in a 100-level mathematics course when teaching the Pythagorean Theorem.
2. Voigt (1996) and Richards (1996) subscribe to subtly different notions of the word "negotiation" within a mathematics classroom. Others (Cobb, Jaworski, & Presmeg, 1996) have distinguished these two notions as "implicit negotiation" (Voigt) and "explicit negotiation" (Richards). Explain in detail each of these two types of negotiation, making a clear distinction between the two. Then discuss the role that negotiation plays in the development of classroom social norms, sociomathematical norms, and classroom mathematical practices as described by Cobb and Yackel (1996).
3. In the literature it is clear that what a mathematician considers to be a valid proof is not well-defined. Consider the attached vignette of a 4th grade class' discussion about even numbers. In this classroom exchange, the students end up debating two seemingly different definitions of what it means for a number to be even. How would you characterize the class' justification? Discuss what it means to be a valid proof at different levels of development. Keep in mind that your response is not intended to be solely an opinion, but your position needs to be backed by the literature you have read on proof.

Section II:

1. Three of the main learning theories in educational psychology are Piaget's Constructivism, Vygotsky's Socio-Culturalism, and Skinner's Behaviorism. Discuss each theory and how they explain learning and motivation.
2. There are several takes on intelligence within the field of educational psychology. For this question, discuss the traditional view of intelligence as measured by IQ tests as well as at least one other view of intelligence. In your explanation, discuss the ways in which these views of intelligence can be used to inform teaching.
3. Describe the current model of human memory and its relation to cognition as outlined in Ormrod. Also, discuss ways in which we can use an understanding of this model to increase student learning.

Vignette: An Odd Thing About Evens

Mrs. Jacobs was teaching a lesson on probability to her 4th grade class. In the process of analyzing a tree diagram and listing possible outcomes for the rolling of a pair of dice, she asked her students what it meant for a number to be even. The following discussion ensued.

Mrs. Jacobs: If we want to look at the probability of getting an even outcome, what can happen on each die?

Fran: Can we get even-even or odd-odd?

Mrs. Jacobs: What do you mean by that, Fran?

Fran: You know, an even on both or an odd on both.

Mrs. Jacobs: Are there any other ways you can get an even number for an outcome?

Tim: How do you know a number's even other than just knowing… You know, that 2, 4, 6, 8, and so on are evens? How can you tell what ways you can get'em from two different dice?

Mrs. Jacobs: That's a good question. How do we know that a number is even?

Evan: I always thought that a number was even if you can break it up into two things that are the same.

Mrs. Jacobs: Tell me more. What do you mean by that?

Evan: Well, you know, 10 is even since it can be 5+5. You know, the same number added to itself twice.

Mrs. Jacobs: That's a great way to look at it.

Kate: Yeah, but that's not the way I learned it.

Mrs. Jacobs: What do you mean, Kate?

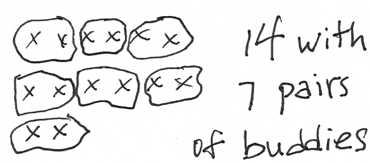
Kate: Well, I always thought a number was even if everyone had a buddy.

Mrs. Jacobs: A buddy?

Kate: Yeah, let's say you have a number like 14. If we have 14 things we can group them in buddies so that everyone has a buddy and no one is left without one.

Mrs. Jacobs: Can you show us?

Kate: (Kate goes to the board and sketches the following)



Kate: See everyone has a buddy and no one is left out.

Mrs. Jacobs: Oh I see. That's interesting. So who's definition is right—Evan's even or Kate's even?

Paul: I like Evan's. I think his is right.

Mia: Kate's is the way I learned it.

Mrs. Jacobs took a vote from the class and Kate's definition won (10 to 8, but it appeared to be based mostly on whether the student was a closer friend to either Evan or Kate).

Mrs. Jacobs: I'm gonna throw out a crazy idea and see what you think of it. Could they both be right?

Ken: There can only be one right answer. This is math.

Mrs. Jacobs: Oh really. Let's see. How can we think about each one? [Pauses] I've got an idea. OK, there are 18 of you here today, right, that's an even number. So I want 9 of you to go to the wall on that side of the room (points to the left) and 9 of you to go to the other side of the room (points to the right). OK, go. (The students move to each side of the room.)

Mrs. Jacobs: Alright, who's definition is this?

LaShawn: It's Evan's even. That sounds funny.

Mrs. Jacobs: Who can explain how this is Evan's even?

Fran: It's like Evan said. He said 10 was even since it can be 5+5…the same number added twice. We have 9+9 because we got 9 on each wall.

Mrs. Jacobs: Good. Now can anyone come up with a way to go from where we are right now to get Kate's buddy definition? (The students think for a minute or two.)

LaShawn: I got it. Take you and you (she points to one person on each side of the room). You're buddies. Now you and you (points again to one person on each side of the room), you're buddies. That'll work.

Mrs. Jacobs: But how do you know no one is left over?

LaShawn: Duh!! We started with the same number on both sides. Remember, that was the way Evan said it.

Mrs. Jacobs: Great job. You just showed how you can take Evan's definition and get Kate's, but can you go the other way?

Fran: Well, duh! If everybody's buddied up, just say, OK (points to a pair of students who are buddies) you go to that side and you go to the other side. (The two students each move to opposite sides of the room). Keep doin' that for all the buddies.

Mrs. Jacobs: But how do you know you will end up with same number of people on each side like in Evan's even? [The class laughs]

Fran: Well, there's a person from each pair on each side so there has to be the same number of each on each wall.

Mrs. Jacobs: Great job! You just showed that given either definition, you can always get the other. In a case like this, we say that these definitions are "logically equivalent". All that really means is that it doesn't matter which definition we use, they are basically the same.

Mrs. Jacobs then continues with her lesson on probability.