Mathematics Education Qualifying Exam
August 2022

Instructions: The following questions constitute the Mathematics Education Qualifying Exam for August of 2022. The questions are separated into two sections; the first section is based on MTH 761 and the second section is based on MTH 762. You must answer all questions in both Section I and Section II. You must pass each section with a 70% score in order to pass the exam. You have four hours to complete this exam. Remember to save your work frequently. Please type your responses directly into this document unless directed otherwise.

Section I: (Answer both questions)

1. Larsen (2013) and Larsen & Lockwood (2013) describe the use of guided reinvention as a means to develop conceptual understanding in students. In these particular cases, they explored using this technique to develop the concepts of group, isomorphism, and quotient groups.

    a. Describe the general process and goals of guided reinvention providing some illustrations of how it is used.

    b. Mejía-Ramos & Weber (2019) document the lack of use of visual representations in linear algebra thinking in comparison to other areas of mathematics. Mejía-Ramos & Weber suggest that this is due to the fact that students are less familiar with visual representations of core concepts in linear algebra. Lapp, Nyman, & Berry (2010) found that the concepts of eigenvalues and eigenvectors are the most disconnected from other concepts in an undergraduate linear algebra course. Note that for many students, their understanding consists of merely computing these mathematical objects via a memorized process of computing determinates of $A - \lambda I$ where A is the transformation matrix and I is the identity matrix since the desire is to replace a matrix transformation with a scalar one. For this part of the question, you will develop a lesson plan for teaching the ***concepts*** of eigenvectors and eigenvalues using the construct of guided reinvention while integrating visual representations with the symbolic form using the file, *TransformPointsEigenVc.tns* provided on the calculator. Recall that you used this file in MTH 761 when we discussed these topics. Your lesson plan will use the format, **Launch-Explore-Summarize**, from MTH 761 and incorporate teacher and student actions along with rationale for your pedagogical decisions. Lesson plan sheets are provided and you should write neatly on these sheets. If you need more sheets for any of the sections of your lesson, please ask and more will be provided. In the rationale column, please explain and cite any relevant research supporting your decisions. Be certain to provide both the questions you would ask students and possible responses you anticipate from them.

2. As we discussed in MTH 761, when we look at facilitating classroom discourse, we can categorize this discourse into 5 routines as follows:

   1. Re-voicing students' contributions
   2. Orienting students to each other
   3. Pressing for explanations
   4. Connecting students' ideas
   5. Making the structure of the discourse visible

Consider the written vignette provided (The Road Trip) describing a class discussion of a position vs time graph. Give your responses to the two questions that follow.

   a. The teacher should use these discourse routines to focus on the students' thinking and work to establish and support productive mathematical discourse. In what ways did the teacher in the vignette (Sean) succeed or fail to accomplish these goals? Explain your thoughts citing specific instances. Particularly, in looking at the discourse routines listed above, identify where in the vignette the teacher employed them. Be specific and cite instances along with how the student(s) responded.

   b. Discuss the types of questions asked by the teacher and the responses they elicited from the students. In particular, relate these discourse routines to Kaput, Blanton, & Moreno's (2008) model for the development of symbolic meaning. How did the teacher's management of discourse help/hinder the development of symbolic meaning?

Section II: (Answer both questions)

For both of these questions, we expect your response to be detailed and for you to use evidence from the literature to support your claims.

1. In the attachment and on the USB flash drive, you will find the Mejía-Ramos & Weber (2019) article, which we read and discussed in MTH 762. For this study:
    a. Identify what type of study this is (quantitative, qualitative, mixed methods). How do you know? That is, what characteristics of the study show the methodology?
    b. Identify, name, and describe at least two strengths of this study's methods. Be sure to draw on what we learned about mathematics education research in class.
    c. Identify, name, and describe at least one weakness of this study's methods. Be sure to draw on what we learned about mathematics education research in class.
    d. What is one change you would make to the study to fix the weakness you described in Part d?
    e. Identify, name, and describe at least one reliability or validity procedure used in this study, again drawing on what we learned in class.

2. For your answers to this question, please cite articles we read in 762 (or that you read for one of your projects) as examples.
    a. In considering quantitative mathematics education research, what is reliability and what is validity? Define each term and give an example from a research article.
    b. We defined and discussed the following terms in class. Pick three of these and define them and then give an example from an article from a research article.
        i.   Predictive validity
        ii.   Face validity
        iii. Content validity
        iv.   Construct validity
        v.   Test-retest correlation
        vi.   Alternate form method
        vii. Split halves method
        viii. Cronbach's alpha